11.4 Surface Area and Volume

I. Surface Area and Volume of any Polyhedron
   A. Surface Area – sum of the area of its faces. 
      (Enough paper to wrap it)

   B. Volume – the measure of space taken up by a solid in three-dimensional space. (box of chocolates 2 layers)

II. Prisms and Cylinders
   A. Surface Area

      **Formula:** Surface Area of a Right Prism or Right Cylinder
      pages 722
      Let a right prism or cylinder have height $h$, bases of area $B$, and let $p$ be the perimeter of each base. Then the surface area, $SA$, is given by:
      
      \[ SA = 2B + ph \]

      **Definition:** If the lateral faces of a prism are all rectangles it is a right prism. Page 641

   B. Volume

      1. RIGHT

      **Formula:** Volume of a Right Prism or a Right Cylinder
      pages 726
      Let a right prism or right cylinder have height $h$ and a base of area $B$. Then its volume, $V$, is given by
      
      \[ V = Bh \]

      2. GENERAL

      **Formula:** Volume of a General Prism or Cylinder pages 728
      A prism or cylinder of height $h$ and base area $B$ has volume
      
      \[ V = Bh \]

      3. Oblique

      **Formula:** Volume of a General Prism or Cylinder pages 728
      A prism or cylinder of height $h$ and base are $B$ has volume
      
      \[ V = Bh \]
11.4 Surface Area and Volume

EXAMPLE 1: Find how much paper it would take to wrap the briefcase (ignore the handle)? How many cubic inches of space does it provide? 3 inches wide by 12 inches high by 17 inches long.

EXAMPLE 2: A can of soda pop is about 9.5 cm tall and has a diameter of about 5.5 cm. How much aluminum is needed to make a can of soda pop?

III. Pyramids and Cones
A. Surface Area
1. Pyramid

Formula: Surface Area of Right Regular Pyramid page 727
Let a right regular pyramid have slant height \( s \), and a base of area \( B \) and perimeter \( p \). Then the surface area, \( SA \), of the pyramid is given by the formula

\[
SA = B + \frac{1}{2} ps
\]

Example 3: A pyramid has a square base 10 cm on a side. The edges that meet at the apex have length 13 cm. Find the slant height of the pyramid, and then calculate the total surface area (including the base) of the pyramid.
11.4 Surface Area and Volume

2. Cone

**Formula:** *Surface Area of a Right Circular Cone page 725*

Let a right circular cone have slant height $s$ and a base of radius $r$. Then the surface area, $SA$, of the cone is given by the formula

$$SA = r^2 + rs$$

**EXAMPLE 4:** An ice cream cone has a diameter of 2.5 inches and slant height of 6 inches. What is the lateral surface area of the cone?

B. Volume

**Formula:** *Volume of a Pyramid or Cone page 730*

The volume, $V$, of a pyramid or cone of height $h$, and base of area $B$ is given by

$$V = \frac{1}{3} Bh$$

**Example 5:** A pyramid has a square base 10 cm on a side. The edges that meet at the apex have length 13 cm. Find the volume of that pyramid.
11.4 Surface Area and Volume

IV. Sphere
1. Surface Area

**Formula:** Surface Area of a Sphere page 732
The surface area of a sphere of radius \( r \) is given by the formula

\[
SA = 4\pi r^2
\]

**Example 6:** If the diameter of the sphere is 6 mm find its surface area.

2. Volume

**Formula:** Volume of a Sphere page 731
The volume, \( V \), of a sphere of radius \( r \) is given by the formula

\[
V = \frac{4}{3}\pi r^3
\]

**Example 7:** Find the Volume of the above Sphere from example 6.

**Example 8:** An ice cream cone is 5 inches high (height) and has an opening 3 inches in diameter. If filled with ice cream and given a hemispherical (half a sphere) top, how much ice cream is there?
11.4 Surface Area and Volume

V. Similarity

Theorem: *The Similarity Principle of Measurement page 734*

Let Figures I and II be similar. Suppose some length dimension of Figure II is \( k \) times the corresponding dimension of Figure I; that is, \( k \) is the scale factor. Then:

1. *any* length measurement – perimeter, diameter, height, slant height, and so on – of Figure II is \( k \) times that of the corresponding length measurement of Figure I;
2. *any* area measurement – surface area, area of a base, lateral surface area, and so on – of Figure II is \( k^2 \) times that of the corresponding area measurement of Figure I;
3. *any* volume measurement – total volume, capacity, half-full, and so on – of Figure II is \( k^3 \) times the corresponding volume measurement of Figure I.

**EXAMPLE 9:**

Television sets are measured by the length of the diagonal of the rectangular screen. How many times larger is the screen area of a 40-inch model than a 13-inch table model?