5.1 Representation of Integers

- Natural numbers? Whole numbers?
- Integers
- Integers are used to describe:
  1. debits & credits
  2. profits & losses
  3. changes in prices
  4. temperature
  5. Golf under par (-18)

Example 1

Example 2
5.1 Representation of Integers

DEF The Integers page 284
- The positive integers are the natural numbers.
- The negative integers are the numbers -1, -2, -3, ...
- where \(-r\) is defined by the equality
  \(s + (-s) = (-s) + s = 0\)
- The integer 0 is neither positive nor negative and
  has the property \(0 + n = n + 0 = n\) for every
  integer \(n\). The integers consist of the positive
  integers, the negative integers, and zero.

5.1 Representation of Integers

Mail-Time
- Representation of Integers
  - "Postman Delivers"
  - Checks +
  - Brings addition
  - Bills -
  - Takeaway subtraction
- Are we better Off
- + By how much?
- Or worse off?
- - by how much?

1. Postman delivers 1 check of $300.
2. Postman delivers 1 bill $25.
3. Postman brings 6 bills
   for $3.
4. Postman takes away 1
   bill for $3.
5. Postman takes away 3
   bills for $4 each.

5.1 Representation of Integers

THM. The Negative of the Negative of
an Integer page 289
- For every integer \(n\), \(-(-n) = n\)

DEF Absolute Value of an Integer
- Distance from zero, always positive
5.2 Addition and Subtraction of Integers

- Remember
  \[ a = n(A), \ b = n(B), \ \text{and} \ A \cap B = \emptyset, \ \text{then} \ a + b \ \text{was defined as} \ n(A \cup B). \]
- Ex. 1
- \[ 9 + (-3) = 5 \]

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5.2 Addition and Subtraction of Integers

- THM: Adding Integers pg. 296
  - Let \( m \) and \( n \) be positive integers so that \(-m\) and \(-n\) are negative. Then the following are true:
  - \((-m) + (-n) = -(m + n)\)
  - If \( m > n \), then \( m + (-n) = m - n \)
  - If \( m < n \), then \( m + (-n) = -(n - m) \)
  - \( n + (-n) = (-n) + n = 0 \)

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5.2 Addition and Subtraction of Integers

- THM: Properties of the Addition of Integers page 297
  - Let \( m, n, \) and \( r \) be integers. Then the following hold.
  - Closure Property \( m + n \) is an integer
  - Commutative Property \( m + n = n + m \)
  - Associative Property \( m + (n + r) = (m + n) + r \)
  - Additive identity \( 0 + m = m = 0 \)
  - Additive inverse \( (-m) + m = m + (-m) = 0 \)
5.2 Addition and Subtraction of Integers

- Draw a number-line diagram to illustrate each of these calculations.
  - (-3) + 7
  - (-5) + (-2)
  - 3 + (-6)
  - 5 + (-5)

5.2 Addition and Subtraction of Integers

- THM: The Law of Trichotomy page 331
  If \( a \) and \( b \) are any two integers, then precisely one of these three possibilities must hold:
  \[ a < b \quad \text{or} \quad a = b \quad \text{or} \quad a > b \]
  Example:
  a. \( 15 ? 101 \)        c. \( 3 ? (-3) \)
  b. \( -8 ? -9 \)          d. \( 0 ? -8 \)

5.2 Addition and Subtraction of Integers

- Subtraction using colored counters page 302
  - \((-3) - 7 = -10\)
    - You have \((-3)\) until you remove 7 black, then you have \((-10)\)
5.2 Addition and Subtraction of Integers

- $5 - (-4) = 9$ or $5 = (-4) + 9$

THM: Closure Property for the Subtraction of Integers page 306

The set of integers is closed under subtraction.

5.3 Multiplication and Division of Integers

- Multiplication is repeated ADDITION
- THM: THE RULE OF SIGNS page 317
  - Let $m$ and $n$ be positive integers so that $-m$ and $-n$ are negative integers. Then the following are true:
    
    \[
    m \cdot (-n) = -(mn) \\
    (-m) \cdot n = -(mn) \\
    (-m) \cdot (-n) = mn \\
    a \cdot 0 = 0 \cdot a = 0 \text{ for any integer } a
    \]
5.3 Multiplication and Division of Integers

| THM: Multiplication Properties of Integers page 319 |
| Closure Property |
| Commutative Property |
| Associative Property |
| Multiplicative Property of One |
| Multiplicative Property of Zero |
| **GIVE AN EXAMPLE OF EACH OF THE ABOVE PROPERTIES.** |

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**Example:** Describe a mail-time situation that illustrates -44.

- a. At mail-time, you are delivered a check for $44.
- b. At mail-time, you are delivered a bill for $44.
- c. At mail-time, you are delivered a check for $44 and a bill for $44.

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**Example:** The letter carrier takes away 4 checks for $16 each.

- a. $4 + 16$
- b. $4 \times 16$
- c. $(-4) \times 16$
- d. $4 \times (-16)$
5.3 Multiplication and Division of Integers

- THM: Rule of Signs for Division of Integers page 322
  - Let \( m \) and \( n \) be positive integers so that \( -m \) and \( -n \) are negative integers and suppose that \( n \) divides \( m \). Then the following are true:
    - \( m ÷ (-n) = -(m ÷ n) \)
    - \( (-m) ÷ n = -(m ÷ n) \)
    - \( (-m) ÷ (-n) = m ÷ n \)

6.1 The Basic Concepts of Fractions and Rational Numbers

A. *Pairs to represent & check for rational number comprehension.* (activity)
1. Size whole parts
2. What is \( \frac{1}{2} \) of \( \frac{2}{3} \)?
   - Leads into fraction multiplication.
3. How many \( \frac{1}{8} \) in \( \frac{2}{7} \)?
   - Leads into fractions division.
6.1 The Basic Concepts of Fractions and Rational Numbers

4. What is \( \frac{2}{4} + \frac{1}{4} - \frac{1}{3} \)?
   - Leads into like and unlike fractions in addition.

5. What is \( \frac{1}{3} - \frac{1}{4} \)?
   - Leads in to like and unlike fractions in subtraction.

---

6.1 The Basic Concepts of Fractions and Rational Numbers

- **DEF. Fractions page 347**
  - First introduce in measurement problems to express a quantity less than a whole unit.

- The Number Line Model page 349
  - Game “Fraction Wars”

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6.1 The Basic Concepts of Fractions and Rational Numbers

- **C. Fraction Strips**

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6.1 The Basic Concepts of Fractions and Rational Numbers

PROPERTY The Fundamental Law of Fractions
Let \( \frac{a}{b} \) be a fraction. Then
\[
\frac{a}{b} = \frac{an}{bn}, \text{ for any integer } n \neq 0.
\]
Example
\[
\frac{1}{2} = \frac{1 \cdot 3}{2 \cdot 3} = \frac{3}{6}
\]

6.1 The Basic Concepts of Fractions and Rational Numbers

THM The Cross-Product Property of Equivalent Fractions
The fractions \( \frac{a}{b} \) and \( \frac{c}{d} \) are equivalent if and only if, \( ad = bc \).
Example
a) \[
\frac{23}{47} = \frac{2231}{4559}
\]

6.1 The Basic Concepts of Fractions and Rational Numbers

DEF. Fractions in Simplest Form
A fraction \( \frac{a}{b} \) is in simplest form if \( a \) and \( b \) have no common divisor larger than 1 and \( b \) is positive.
Method 1: Divide successively by common factors.
EX.
\[
\frac{240}{360}
\]
DEF. *Fractions in Simplest Form (cont’d)*

**Method 2:** Divide \( a \) and \( b \) by GCD \( (a, b) \)

\[ \frac{28}{40} \]

**Method 3:**
Divide by the common factors in the prime factorization of \( a \) and \( b \).

\[ \frac{-45}{135} \]

**Method 4:** Use a fraction calculator.

\[ \frac{117}{-468} \]

**Common Denominators**

\[ \frac{3}{4} + \frac{5}{8} + \frac{2}{3} \]

\[ \frac{15}{34} + \frac{12}{51} \]
6.1 The Basic Concepts of Fractions and Rational Numbers

**DEF. Rational Numbers (end or repeat)**
A rational number is a number that can be represented by a fraction \( \frac{a}{b} \), where \( a \) and \( b \) are integers, \( b \neq 0 \). Two rational numbers are equal if, and only if, they can be represented by equivalent fractions.

**DEF. Order Relation on the Rational Numbers**
Let two rational numbers be represented by the fractions \( \frac{a}{b} \) and \( \frac{c}{d} \), with \( b \) and \( d \) positive. Then \( \frac{a}{b} \) is less than \( \frac{c}{d} \) written \( \frac{a}{b} < \frac{c}{d} \), if, and only if \( ad < bc \).

### Comparing Rational Numbers

Example:

1. \( \frac{3}{4} \) and \( \frac{2}{5} \)
2. \( \frac{15}{29} \) and \( \frac{6}{11} \)
3. \( \frac{2106}{7047} \) and \( \frac{234}{783} \)
4. \( -\frac{10}{13} \) and \( -\frac{22}{29} \)

6.2 The Arithmetic of Rational Numbers

**DEF: Addition of Rational Numbers** page 365 (LIKE FRACTIONS)
Let two rational numbers be represented by fractions \( \frac{a}{b} \) and \( \frac{c}{d} \) with a common denominator. Then their sum of the rational numbers is given by

\[
\frac{a}{b} + c = \frac{a + b}{b}
\]
6.2 The Arithmetic of Rational Numbers

Draw
\[
\frac{1}{4} + \frac{2}{4}
\]

6.2 The Arithmetic of Rational Numbers

Addition of UNLIKE FRACTIONS

Draw
\[
\frac{2}{3} + \frac{1}{2}
\]

6.2 The Arithmetic of Rational Numbers

Example

1. \[
\left( \frac{3}{4} \cdot \frac{5}{6} \right) - \frac{2}{3}
\]
2. \[
\frac{3}{8} + \frac{-7}{24}
\]
6.2 The Arithmetic of Rational Numbers

- Draw \( \frac{3}{8} \) what is it as an improper fraction?
- Give a mixed number for \( \frac{355}{133} \)
- Give a mixed number for \( \frac{-15}{4} \)
- Compute \( \frac{3}{4} + \frac{2}{5} \)

6.2 The Arithmetic of Rational Numbers

- Which of the following are Proper Fractions?

\[
\begin{align*}
\frac{1}{5} &\quad \frac{8}{7} &\quad \frac{2}{2} &\quad \frac{4}{7} &\quad \frac{15}{17}
\end{align*}
\]

6.2 The Arithmetic of Rational Numbers

- DEF: Subtraction of Rational Numbers
  page 371

Let \( \frac{a}{b} \) and \( \frac{c}{d} \) be rational numbers.

Then \( \frac{a}{b} - \frac{c}{d} = \frac{e}{f} \) if, and only if,

\[
\frac{a}{b} - \frac{c}{d} = \frac{e}{f}
\]
6.2 The Arithmetic of Rational Numbers

Example
1. \[ \frac{4}{5} \div \frac{2}{3} \]
2. \[ 8 \frac{1}{3} \div 4 \frac{3}{5} \]

DEF: Multiplication of Rational Numbers
page 373
Let \( \frac{a}{b} \) and \( \frac{c}{d} \) be rational numbers. Then their product is given by:
\[ \frac{a}{b} \cdot \frac{c}{d} = \frac{ab}{cd} \]

Examples (decimal answers are not acceptable.)
1. \[ \frac{5}{8} \div \frac{2}{3} \]
2. \[ 3 \frac{1}{7} \div 5 \frac{1}{4} \]
6.2 The Arithmetic of Rational Numbers

THM: The Invert and Multiply Algorithm for Division of Fractions and Rational Numbers.
Page 378
\[
\frac{a}{b} : \frac{c}{d} = \frac{a}{d} \cdot \frac{b}{c}, \text{ where } c \neq 0.
\]

DEF: Reciprocal of a Rational Number
- The reciprocal of a nonzero rational number \( \frac{c}{d} \) is \( \frac{d}{c} \).

Example
1. \( \frac{3}{4} \div \frac{1}{8} \)
2. \( \frac{4}{6} \div \frac{1}{3} \)

FRACTION WARS

Purpose: Reinforce estimation and comparison of fractions in a game format.
- Play 10 rounds of each goal with your partner.
- GOAL 1: Form a fraction by placing the card with the smaller number in the numerator. Player with the smaller fraction is the winner.
- GOAL 2: Form a fraction by placing the card with the larger number in the numerator. Player with the larger fraction is the winner.
- GOAL 3: Place the first card in the numerator and the second in the denominator. Player with the fraction whose value is closest to 2 is the winner.
6.3 The Rational Number System

- Density page 391
  - There are infinitely many rational numbers between two rational numbers.
  - Find at least two fractions between:

\[
\frac{5}{6} \text{ and } \frac{3}{7} \quad \frac{12}{14} \text{ and } \frac{10}{15}
\]

CONCEPT MAPS

- Complete the following in your groups:
  - Chapter 5. Whole-Number Arithmetic Concept Map
  - Chapter 6. Fraction Concept Map